

Kant on the synthetic character of mathematics

Letter to J. Schultz, 25 Nov., 1788

...I can form a concept of one and the same quantity by means of many different additions and subtractions (notice that both of these processes are syntheses, however). Objectively, the concepts I form are identical (as in every equation). But subjectively, depending on the type of composition that I think in order to arrive at that concept, they are very different. So that at any rate my judgment goes beyond the concept I get from the synthesis, in that the judgment substitutes another concept (simpler and more appropriate to the construction) in place of the first concept, though it determines the same object. Thus I can arrive at a single determination of a quantity by means of $3+5$, or $12-4$, or 2×4 , or 2^3 , namely 8. But my thought ' $3+5$ ' did not include the thought ' 2×4 '. Just as little did it include the concept '8', which is equal in value to any of these...

[I]f I regard $3+4$ as the setting of a problem, namely to find a third number (7) such that the one number will be seen as the completion-to-a-total of the other, the solution is found by means of the simplest operation, requiring no special prescription, namely, by the successive addition that the number 4 proposes simply as a continuation of the counting up to 3. The judgment " $3+4=7$ " does seem to be a purely theoretical judgment, and objectively regarded, that is what it is; but subjectively, the sign '+' signifies the synthesis involved in getting a third number out of two other numbers, and it signifies a task to be done, requiring no instruction or proof. Consequently the judgment is a postulate. Now assuming it were an analytic judgment, I would have to think exactly the same things by ' $3+4$ ' as by '7', and the judgment would only make me more clearly conscious of what I thought. But since $12 - 5$ yields a number (7) that is actually the same number I thought when I was adding $3+4$, it follows, according to the principle 'things equal to the same thing are equal to each other', that when I think '3 and 4' I must at the same time be thinking '12 and 5'. And this does not jibe with my own awareness.

All analytic judgment by means of concepts have this characteristic: they can represent a predicate only as a constituent concept contained in the subject-concept. In the case of definitions, both concepts must be reciprocal. But in an arithmetic judgment, namely, an equation, both concepts must be absolutely reciprocal and objectively identical, for example, the concept ' $3+4$ ' and '7'. In the problem conjoin 3 and 4 in one number, the number 7 must arise not out of an analysis (taking apart) of the constituent concepts but rather by means of a construction, that is, synthetically (putting together). This construction, a single counting up in an a priori intuition [time], exhibits the concept of the conjunction of two numbers. Here we have the construction of the concept of quantity rather than that of a quantum [a whole composed of parts]. For the idea that the conjoining of 3 and 4, as distinct quantitative concepts, could yield the concept of *one* magnitude was only a thought. The number 7 is thus the exhibition of this thought in an act of counting together.

Time, as you correctly notice, has no influence on the properties of numbers (considered as pure determinations of quantity)... The science of numbers, notwithstanding the succession that every construction of quantity requires, is a pure intellectual synthesis, which we represent to ourselves in thought. But insofar as specific quantities (quanta) are to be determined in accordance with this science, they must be given to us in such a way that we can grasp their intuition successively; and thus this grasping is subjected to the time condition. So that when all is said and done, we cannot subject any object other than an object of a possible *sensible* intuition to quantitative, numerical assessment, and it thus remains a principle without exception that mathematics can be applied only to *sensibilia*. The magnitude of God's perfection, of duration, and so on, could only be expressed by means of the [idea of the] *totality* of reality; it could not possibly be represented by means of numbers, supposing someone wanted to measure even a merely intelligible unity.